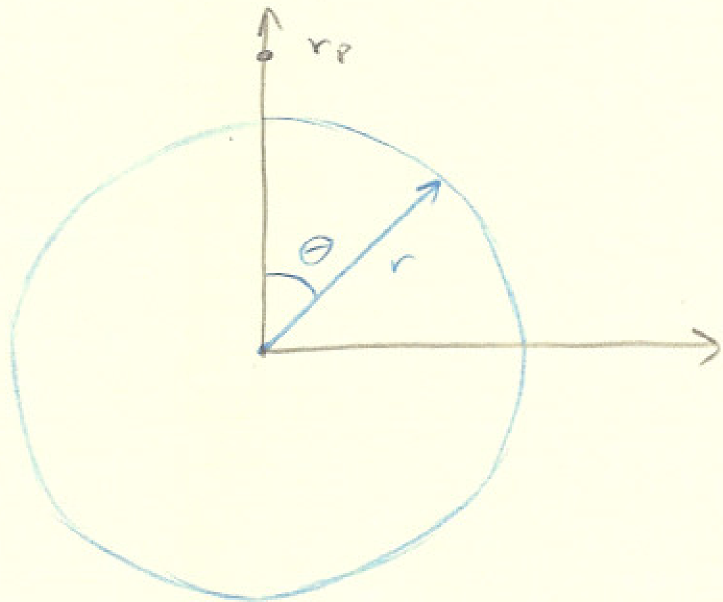
Spherical Shell



note The magnetude of $\vec{E}$ at any point located a distance "r" from the origin must be the same

Divide the surface into small peices.

$$dA_s = r^2 \sin\theta_s \, d\theta_s \, d\phi_s$$

We know that

$$A = r^2 \int_0^{\pi} d\theta_s \sin\theta_s \int_0^{2\pi} d\phi = 4\pi r^2$$

$$dq = \sigma_0 \, dA_s$$

$$\vec{E} = \frac{dq}{4\pi\varepsilon_0} \frac{\vec{r_p} - \vec{r_s}}{|\vec{r_p} - \vec{r_s}|^3}$$

$$\vec{r}_p - \vec{r}_s = -\hat{x}\, r\sin\theta_s\cos\phi - \hat{y}\, r\sin\theta_s\sin\phi_s + \hat{z}(z_p - R\cos\theta_s)$$

$$|\vec{r}_p - \vec{r}_s| = \left[r^2\sin^2\theta_s\cos^2\phi + r^2\sin^2\theta_s\sin^2\phi + (z_p - r\cos\theta_s)^2\right]^{1/2}$$

$$= \left[r^2\sin^2\theta + (z_p - r\cos\theta_s)^2\right]^{1/2}$$

$$\left.\begin{array}{l} E_x = 0 \\ E_y = 0 \end{array}\right\} \text{b/c sym}$$

$$E_z = \frac{\sigma_0 r^2}{4\pi\varepsilon_0}\int_0^{\pi} d\theta \sin\theta \int_0^{2\pi} d\phi\, \frac{(z_p - r\cos\theta)}{(r^2\sin\theta_s + (z_p - r\cos\theta)^2)^{3/2}}$$

$$E_z = \frac{(2\pi) r^2}{4\pi\varepsilon_0}\int_0^{\pi} \frac{\sin\theta\,(z_p - r\cos\theta)\,d\theta}{(r^2\sin\theta_s + (z_p - r\cos\theta)^2)^{3/2}}$$

$$E_z = \frac{\sigma_0 r^2}{2\varepsilon_0}\left[\frac{r}{z_p}\left(1 - \frac{z_p - R}{\sqrt{(R - z_p)^2}}\right)\right]$$